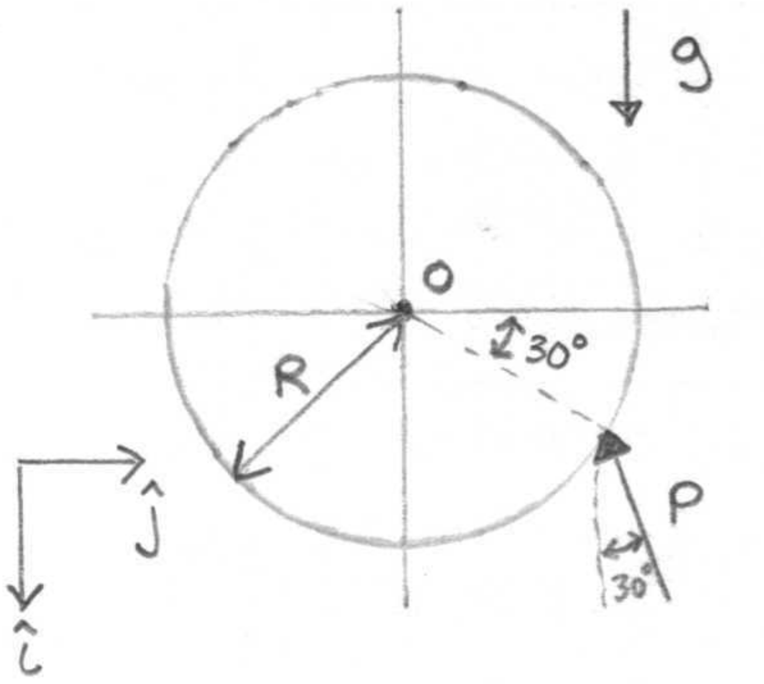
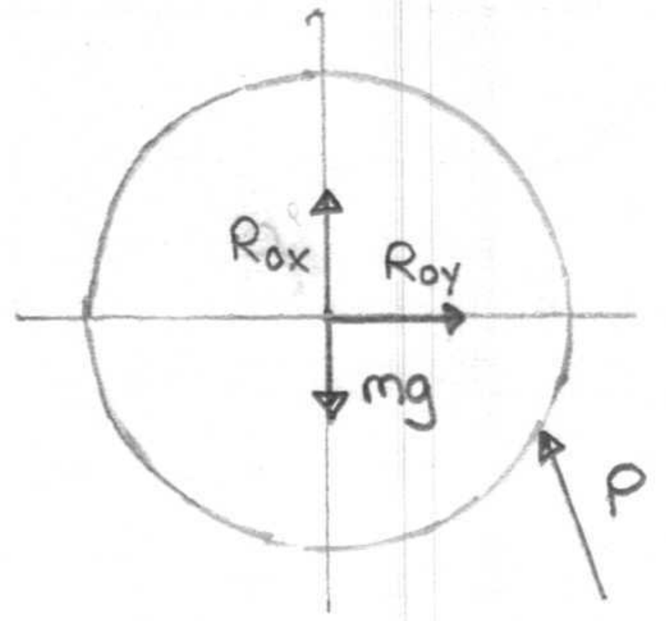
TAM 203
Homework Solutions

Due 4/22/08

13.91

FBD:





Use angular momentum balance about O to find $\dot{\omega}$, given $\theta = 30°$

$$\Sigma \vec{M}_{/O} = \dot{\vec{H}}_{/O} = \dot{\omega} I_O \hat{k} = \tfrac{1}{2} m R^2 \dot{\omega} \hat{k}$$

$$\Sigma \vec{M}_{/O} = \vec{r}_P \times \vec{P} = R(\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}) \times P(-\cos\theta\,\hat{\imath} - \sin\theta\,\hat{\jmath})$$

$$= RP\hat{k}(-\sin^2\theta + \cos^2\theta) = RP\cos(2\theta)\,\hat{k}$$

$$\{\Sigma \vec{M}_{/O} = \dot{\vec{H}}_{/O}\} \cdot \hat{k} \rightarrow \tfrac{1}{2} m R^{\cancel{2}} \dot{\omega} = \cancel{R} P \cos(2\theta)$$

$$\therefore \quad \dot{\omega} = \frac{2P}{mR}\cos(2\theta) = P/mR \quad \therefore \boxed{\dot{\omega}_O = \alpha_O = P/mR}$$

Use linear momentum balance to find reactions at O:

$$\Sigma \vec{F} = m\vec{a}_G \text{ OR } mg\hat{\imath} - R_{Ox}\hat{\imath} + R_{Oy}\hat{\jmath} - P(\cos\theta\,\hat{\imath} + \sin\theta\,\hat{\jmath}) = \vec{0}$$

$$\{\} \cdot \hat{\imath} \rightarrow mg - R_{Ox} - P\cos(30°) = 0$$

$$\therefore \boxed{R_{Ox} = mg - P/2}$$

$$\{\} \cdot \hat{\jmath} \rightarrow R_{Oy} - P\sin(30°) = 0$$

$$\therefore \boxed{R_{Oy} = \sqrt{3}P/2}$$

13.96

a) $I = \frac{1}{2}MR^2$, Power $P = \dot{E}_k = \frac{d}{dt}\left(\frac{1}{2}I\omega^2\right)$

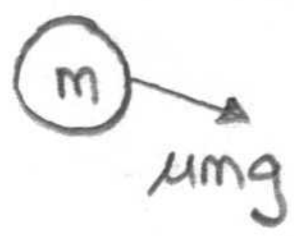
$$\therefore \quad \int P\,dt = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 = Pt + \cancel{c}^{\,0}$$

$$\frac{1}{4}MR^2\omega^2 = Pt \quad \therefore \quad \boxed{\omega = \frac{2}{R}\sqrt{\frac{Pt}{M}}}$$

b) $$\alpha = \frac{d\omega}{dt} = \frac{\cancel{2}}{R}\left(\frac{1}{\cancel{2}}\right)\left(\frac{Pt}{M}\right)^{-\frac{1}{2}}\left(\frac{P}{M}\right) \rightarrow \boxed{\alpha = \sqrt{\frac{P}{MR^2t}}}$$

c) At slip, friction force $F_{fr} = \mu mg$ (magnitude

$$\|\Sigma\vec{F}\| = \|m\vec{a}\|$$



$$\mu \cancel{m} g = \|\cancel{m}(\dot{\vec{\omega}} \times r\hat{e}_r - \omega^2 r\hat{e}_r)\|$$

$$\therefore \quad \mu g = \sqrt{(r\alpha)^2 + (\omega^2 r)^2} = \sqrt{\frac{Pr^2}{MR^2t} + \frac{4r^2}{R^2}\frac{Pt}{M}}$$

Solving, $$\mu g = \sqrt{\frac{Pr^2}{MR^2}\left(\frac{1}{t} + 4t\right)} = \sqrt{\frac{Pr^2}{MR^2}\left(\frac{1+4t^2}{t}\right)}$$

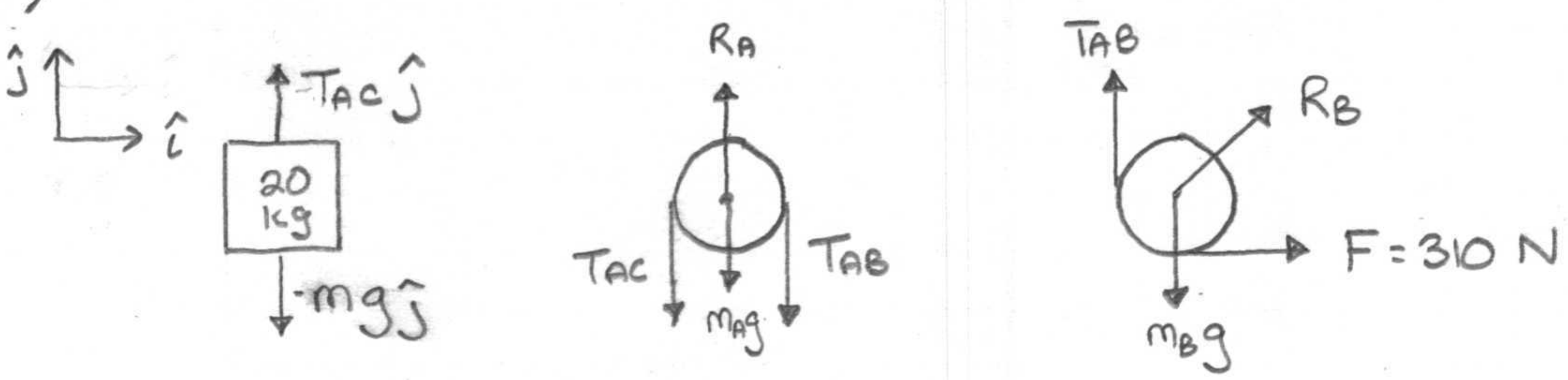
$\therefore$ We have to solve the following for t:

$$\boxed{(\mu g)^2 = \frac{Pr^2}{MR^2t}(1+4t^2)}$$

13.108

Given: $m = 20\ kg$, $m_A = 10\ kg$, $m_B = 5\ kg$

a)



$\{\Sigma \vec{F} = m\vec{a}\}\cdot\hat{j}$ $\qquad$ $\Sigma T = I\alpha$ $\qquad$ $\Sigma T = I\alpha$

$-mg + T_{AC} = m\ddot{y}$ $\qquad$ $\cancel{r_A}(T_{AC} - T_{AB}) = \frac{1}{2} m_A r_A^{\cancel{2}} \frac{\ddot{y}}{\cancel{r_A}}$ $\qquad$ $\cancel{r_B}(F - T_{AB}) = \frac{1}{2} m_B r_B^{\cancel{2}} \frac{\ddot{y}}{\cancel{r_B}}$

$T_{AC} = mg + m\ddot{y}$ (1) $\qquad$ $T_{AC} - T_{AB} = -\frac{1}{2} m_A \ddot{y}$ (2) $\qquad$ $F - T_{AB} = \frac{1}{2} m_B \ddot{y}$ (3)

Equate (1) and (2): $mg + m\ddot{y} = T_{AB} - \frac{1}{2} m_A \ddot{y}$

OR $\frac{1}{2} m_A \ddot{y} + m\ddot{y} = -mg + T_{AB} \rightarrow \ddot{y}\left(\frac{1}{2} m_A + m\right) = -mg + T_{AB}$ (4)

Solve (3) for T_{AB}: $T_{AB} = F - \frac{1}{2} m_B \ddot{y}$

$\therefore$ (4) gives $\ddot{y}\left(\frac{1}{2} m_A + m\right) = -mg + F - \frac{1}{2} m_B$

OR $\ddot{y}\left(\frac{1}{2} m_A + m + \frac{1}{2} m_B\right) = -mg + F$

$$\therefore \ddot{y} = \frac{-(20\ kg)(10.0\ m/s^2) + 310\ N}{\frac{1}{2}(10) + 20 + \frac{1}{2}(5)} = 4.0\ m/s^2$$

We know $\alpha_B = \frac{\ddot{y}}{r_B} = \frac{-4\ m/s^2}{0.2\ m} = \boxed{20\ rad/s^2}$

b) We found this already in (a), $\boxed{\ddot{y} = 4.0\ m/s^2}$

c) We know $T_{AC} = mg + m\ddot{y} = 20kg(10 + 4\ m/s^2)$

$$\boxed{T_{AC} = 280\ N}$$

13.119

a) From longest to shortest period, we may expect:

(c), (d), (a), (b), (e)

b) Use $\sum \vec{M}_{/0} = \dot{\vec{H}}_{/0} = I\dot{\omega}\hat{k}$ OR $\sum M_{/0} = I\dot{\omega}$

(a) $\cancel{M}g(m\sin\theta) + \cancel{M}g(2m\sin\theta) = \cancel{M}(m^2+4m^2)\dot{\omega}$

$3g\cancel{m}\sin\theta = 5m^{\cancel{2}}\dot{\omega}$ OR $\dot{\omega} = \frac{3}{5m}g\sin\theta \approx \frac{3}{5}g\theta$

$\therefore\ \ddot{\theta} - \frac{3g}{5m}\theta = 0$, so $T = 2\pi/\sqrt{\frac{3}{5m}g} = \boxed{2\pi\sqrt{5m/3g}}$

(b) $\cancel{M}g(m\sin\theta) = \dot{\omega}\int_0^{2m}\frac{M}{2m}x^2\,dx = \dot{\omega}\frac{M}{2m}\left(\frac{8m^3}{3}\right) = \frac{4Mm^2}{3}\dot{\omega}$

$\cancel{m}g\sin\theta = \frac{4}{3}m^{\cancel{2}}\dot{\omega}$ OR $\ddot{\theta} - \frac{3g}{4m}\theta = 0 \rightarrow \boxed{T = 2\pi\sqrt{\frac{4m}{3g}}}$

(c) $Mg(2m\sin\theta) - Mg(m\sin\theta) = M(m^2+4m^2)\ddot{\theta}$

$g\cancel{m}\sin\theta = 5m^{\cancel{2}}\ddot{\theta}$ OR $\ddot{\theta} - \frac{g}{5m}\theta = 0 \rightarrow \boxed{T = 2\pi\sqrt{\frac{5m}{g}}}$

(d) $Mg(2\cancel{m}\sin\theta) = 4m^{\cancel{2}}M\ddot{\theta}$

$2g\sin\theta = 4m\ddot{\theta}$ OR $\ddot{\theta} - \frac{g}{2m}\theta = 0 \rightarrow \boxed{T = 2\pi\sqrt{\frac{2m}{g}}}$

(e) $Mg\cancel{m}\sin\theta = m^{\cancel{2}}M\ddot{\theta}$

$g\sin\theta = m\ddot{\theta}$ OR $\ddot{\theta} - \frac{g}{m}\theta = 0 \rightarrow \boxed{T = 2\pi\sqrt{m/g}}$

c) From longest to shortest period: (as expected)

(c), (d), (a), (b), (e)